AP Calculus BC- Unit 1

From Simple Studies: https://simplestudies.edublogs.org and

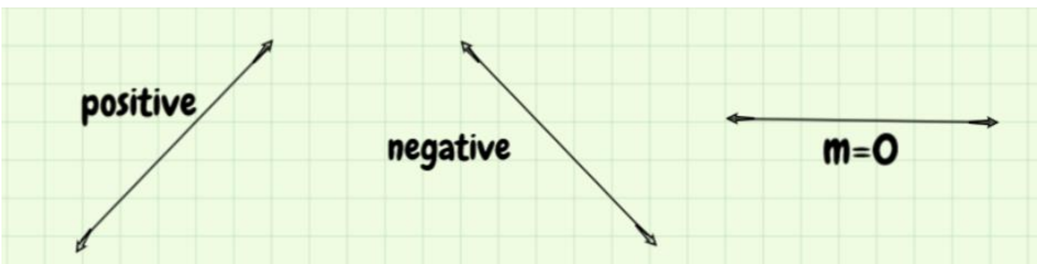
@simplestudiesinc on Instagram

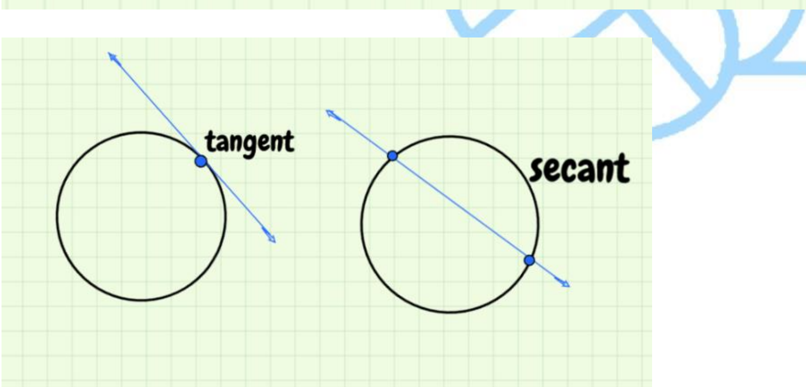
Differentiation: Definition and Fundamental Properties

Chapter 2.1 The derivative and the tangent

Review

Slope= $\frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \frac{\Delta y}{\Delta x}$





Definitions of Derivatives

- Slope of a tangent line is the derivative

Slope is a derivative

$$m=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ (this is the definition)

EX: Find the derivative of the function f(x)=x^2 using the definition.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x-h)^2 \; -x^2}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h)}{h}=2x$$

f'(x)= 2x

Chapter 2.2 Basic Differentiation Rules

Notations for Derivative: f(x), f',y', dy/dx,d/dx[ ]. Derivative in respect to x.

The Constant Rule

$\frac{d}{dx}$[c]=0 *The derivative of any constant is always 0.

Power rule

f(x)= a*x^n

f'(x)= a*nx^{n-1}

Ex: f(x)=5$x^{56} - 1236x^4 + 233.7$

f'(x)=$280x^{53} - 4944x^3$

The Constant Multiple Rule

$\frac{d}{dx}$[cf(x)]= c*f'(x)

Ex: f(x)= 5x^3

f'(x)= 5*3x^2 = 15x^2

The Sum and Difference Rule

$\frac{d}{dx}$[f(x) +- g(x)]= f'(x)+-g'(x)

=$\frac{d}{dx}$[f(x) +- $\frac{d}{dx}$[g(x)]

EX: f(x)=4$x^2 + 3x^6 + 3x^2 - 1$

f'(x)= 4(7x^6) $+ 3(6x^5) + 3(2x) + 0$

=28$x^6 + 18x^5 + 6x$

The derivatives of sin and cos

$\frac{d}{dx}$[sinx]= cosx

$\frac{d}{dx}$[cosx]= -sinx

Derivative of e^x

$$\frac{d}{dx}[e^x] = e^x$$

Chapter 2.3 Product and Quotient Rules

Product Rule

$\frac{d}{dx}$[f(x)*g(x)]= f'(x)*g(x) +f(x)*g'(x)

Quotient Rule

If f and g are differentiable function, then:

$\frac{d}{dx}[\frac{f(x)}{g(x)}]=\frac{f'g - fg'}{g^2}$

Derivatives of trig functions

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

from:https://www.onlinemathlearning.com/image-files/trig-derivatives.png

Normal line

Fancy calculus term for “perpendicular”
Implies that we start finding the tangent slope, and then we find the opposite reciprocal slope to use the normal line

Differentiability of a graph for a point, **requires** the graph to be continuous at the point.

- If a function is NOT continuous then it is Not differentiable
- If a function is continuous, it MAY be differentiable
- If a function is differentiable, it is automatically continuous

Importance: Differentiability→Continuity→limit→Graph/Function
You can “see” if a function is differentiable if you can draw it smoothly
-1 loop whole there is a function that isn’t differentiable (being an asymptote because at one point you will have to lift up the pencil)